# انتگرال[۳]
# بخش ۳

[۳] Sh. Nosrati, 13961208 update 14000502 ES0.

$$
\begin{aligned}
I_{۱۱} &= \int \frac{\sqrt{۱+\sin x}}{\sqrt{۱-\sin x}}\,\mathrm{dx} \\
&= \int \frac{\sqrt{۱+\sin x}}{\sqrt{۱-\sin x}}\frac{\sqrt{۱-\sin x}}{\sqrt{۱-\sin x}}\,\mathrm{dx} \\
&= \int \frac{\cos x}{۱-\sin x}\,\mathrm{dx} \\
&= -\ln(۱-\sin x)+C \\
&= \ln\frac{۱}{۱-\sin x}+C \quad , \quad x \neq ۲k\pi+\frac{\pi}{۲}
\end{aligned}
$$

در حالت معین هم با همان روش عمل می کنیم:

$$
\begin{aligned}
\int_{۰}^{\pi/۶} \frac{\sqrt{۱+\sin x}}{\sqrt{۱-\sin x}}\,\mathrm{dx} &= \ln\frac{۱}{۱-\sin x}\Big|_{۰}^{\pi/۶} \\
&= \ln ۲
\end{aligned}
$$

حاصل انتگرال زیر چیست؟

$$
\int_{۰}^{\pi/۲} \frac{\sqrt{۱+\sin x}}{\sqrt{۱-\sin x}}\,\mathrm{dx}
$$

$$
\begin{aligned}
I_{۱۲} &= \int \frac{۱}{۲+x^{۲}}\,\mathrm{dx} \\
&= \int \frac{۱}{۲(۱+u^{۲})}\sqrt{۲}\,\mathrm{du} \quad ; \quad x=\sqrt{۲}u \implies \mathrm{dx}=\sqrt{۲}\,\mathrm{du} \\
&= \frac{۱}{\sqrt{۲}}\int \frac{۱}{۱+u^{۲}}\,\mathrm{du} \\
&= \frac{۱}{\sqrt{۲}}\arctan u + C \\
&= \frac{۱}{\sqrt{۲}}\arctan \frac{x}{\sqrt{۲}} + C
\end{aligned}
$$

در حالت کلی

$$
\begin{aligned}
\int \frac{۱}{a^{۲}+x^{۲}}\,\mathrm{dx} &= \int \frac{۱}{a^{۲}(۱+u^{۲})}a\,\mathrm{du} \quad ; \quad x=au \implies \mathrm{dx}=a\,\mathrm{du} \\
&= \frac{۱}{a}\int \frac{۱}{۱+u^{۲}}\,\mathrm{du} \\
&= \frac{۱}{a}\arctan u + C \\
&= \frac{۱}{a}\arctan \frac{x}{a} + C
\end{aligned}
$$

$$
\begin{aligned}
I_{۱۳} &= \int \frac{\cos x}{۱+\sin^{۲} x}\,\mathrm{dx} \\
&= \int \frac{۱}{۱+u^{۲}}\,\mathrm{du} \quad ; \quad \sin x = u \implies \cos x\,\mathrm{dx} = \mathrm{du} \\
&= \arctan u + C \\
&= \arctan(\sin x) + C
\end{aligned}
$$

$$
\begin{aligned}
I_{۱۴} &= \int \frac{۱}{۱+\sin^{۲} x}\,\mathrm{dx} \\
&= \int \frac{۱+\cot^{۲} x}{۲+\cot^{۲} x}\,\mathrm{dx} \quad ; \quad \sin^{۲} x = \frac{۱}{۱+\cot^{۲} x} \\
&= -\int \frac{۱}{۲+u^{۲}}\,\mathrm{du} \quad ; \quad \begin{cases} \cot x = u \\ (۱+\cot^{۲} x)\,\mathrm{dx} = -\,\mathrm{du} \end{cases} \\
&= -\frac{۱}{\sqrt{۲}} \arctan \frac{u}{\sqrt{۲}} + C \\
&= -\frac{۱}{\sqrt{۲}} \arctan \frac{\cot x}{\sqrt{۲}} + C
\end{aligned}
$$

$$
\begin{aligned}
I_{۱۵} &= \int \frac{x^{۲}-۱}{x^{۴}+۳x^{۲}+۱}\,\mathrm{dx} \\
&= \int \frac{x^{۲}-۱}{x^{۲}+(x^{۲}+۱)^{۲}}\,\mathrm{dx} \\
&= \int \frac{۱-\frac{۱}{x^{۲}}}{۱+\left(x+\frac{۱}{x}\right)^{۲}}\,\mathrm{dx} \quad ; \quad \begin{cases} x+\frac{۱}{x}=u \\ \left(۱-\frac{۱}{x^{۲}}\right)\mathrm{dx}=\mathrm{du} \end{cases} \\
&= \int \frac{۱}{۱+u^{۲}}\,\mathrm{du} \\
&= \arctan\left(x+\frac{۱}{x}\right)+C
\end{aligned}
$$

حاصل انتگرال زیر چیست؟

$$\int_{۰}^{\infty} \frac{x^{۲}-۱}{x^{۴}+۳x^{۲}+۱}\,\mathrm{dx}$$